

## Preface

We didn't set out to write a new book. The plan was simply to revise our 1991 book, *Thinking Strategically*. But it didn't quite turn out that way.

One model for writing a revision comes from Borges's character Pierre Menard, who decides to rewrite Cervantes's *Don Quixote*. After great effort, Menard's revision ends up being word-for-word identical to the original. However, 300 years of history and literature have passed since *Quixote*, including *Quixote* itself. Although Menard's words are the same, his meaning is now entirely different.

Alas, our original text wasn't *Don Quixote*, and so the revision did require changing a few words. In fact, most of the book is entirely new. There are new applications, new developments in the theory, and a new perspective. So much is new that we decided a new title was called for as well. Although the words are new, our meaning remains the same. We intend to change the way you see the world, to help you think strategically by introducing the concepts and logic of game theory.

Like Menard, we have a new perspective. When we wrote *Thinking Strategically*, we were younger, and the zeitgeist was one of self-centered competition. We have since come to the full realization of the important part that cooperation plays in strategic situations, and how good strategy must appropriately mix competition and cooperation.\*

We started the original preface with: "Strategic thinking is the art of outdoing an adversary, knowing that the adversary is trying to do the same to you." To this we now add: It is also the art of finding ways to cooperate, even when others are motivated by self-interest, not benevolence. It is the art of convincing others, and even yourself, to do what you say. It is the art of interpreting and revealing information. It is the art of putting yourself in others' shoes so as to predict and influence what they will do.

We like to think that *The Art of Strategy* includes this older, wiser perspective. But there is also continuity. Even though we offer more real-life stories, our purpose remains to help you develop your own ways of thinking about the strategic situations you will face; this is not an airport book offering "seven steps for sure strategic success." The situations you face will be so diverse that you will succeed better by knowing some general principles and adapting them to the strategic games you are playing.

Businessmen and corporations must develop good competitive strategies to survive, and find cooperative opportunities to grow the pie. Politicians have to devise campaign strategies to get elected and legislative strategies to implement their visions. Football coaches plan strategies for players to execute on the field. Parents trying to elicit good behavior from children must become amateur strategists—the children are pros.

Good strategic thinking in such numerous diverse contexts remains an art. But its foundations consist of some simple basic principles—an emerging science of strategy, namely game theory. Our premise is that readers from a variety of backgrounds and occupations can become better strategists if they know these principles.

Some people question how we can apply logic and science to a world where people act irrationally. It turns out that there is often method to the madness. Indeed, some of the most exciting new insights have come from recent advances in behavioral game theory, which incorporates human psychology and biases into the mix and thus

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\* Pursuing this line led one of us to write a book on this idea; see Adam Brandenburger and Barry Nalebuff, *Co-opetition* (New York: Doubleday, 1996).

adds a social element to the theory. As a result, game theory now does a much better job dealing with people as they are, rather than as we might like them to be. We incorporate these insights into our discussions.

While game theory is a relatively young science—just over seventy years old—it has already provided many useful insights for practical strategists. But, like all sciences, it has become shrouded in jargon and mathematics. These are essential research tools, but they prevent all but the specialists from understanding the basic ideas. Our main motive for writing *Thinking Strategically* was the belief that game theory is too interesting and important to leave to the academic journals. The insights prove useful in many endeavors—business, politics, sports, and everyday social interactions. Thus we translated the important insights back into English and replaced theoretical arguments with illustrative examples and case studies.

We are delighted to find our view becoming mainstream. Game theory courses are some of the most popular electives at Princeton and Yale, and most other schools where they are offered. Game theory permeates strategy courses in MBA programs, and a Google search for game theory produces more than 6 million pages. You'll find game theory in newspaper stories, op-eds, and public policy debates.

Of course, much of the credit for these developments belongs to others: to the Economics Nobel Prize Committee, which has awarded two prizes in game theory—in 1994, to John Harsanyi, John Nash, and Reinhard Selten and in 2005, to Robert Aumann and Thomas Schelling;\* to Sylvia Nasar, who wrote *A Beautiful Mind*, the best-selling biography of John Nash; to those who made the award-winning movie of the same name; and to all those who have written books popularizing the subject. We might even share a bit of the credit. Since publication, *Thinking Strategically* has sold 250,000 copies. It has been translated into numerous languages, and the Japanese and Hebrew translations have been best sellers.

We owe a special debt to Tom Schelling. His writings on nuclear strategies, particularly *The Strategy of Conflict* and *Arms and Influence*, are justly famous. In fact, Schelling pioneered a lot of game theory in the process of applying it to nuclear conflict. Michael Porter's *Competitive Strategy*, drawing on the lessons of game theory for business strategy, is equally important and influential. An annotated guide to the works of Schelling, Porter, and many others is provided in our Further Reading section.

In this book we do not confine the ideas to any particular context. Instead, we offer a wide range of illustrations for each basic principle. Thus readers from different backgrounds will all find something familiar here. They will also see how the same principles bear on strategies in less familiar circumstances; we hope this will give them a new perspective on many events in news as well as history. We also draw on the shared experience of our readers, with illustrations from, for example, literature, movies, and sports. Serious scientists may think this trivializes strategy, but we believe that familiar examples are an effective vehicle for conveying the important ideas.

The idea of writing a book at a more popular level than that of a course text came from Hal Varian, now at Google and the University of California, Berkeley. He also gave us many useful ideas and comments on earlier drafts. Drake McFeely at W. W. Norton was an excellent if exacting editor for *Thinking Strategically*. He made extraordinary efforts to fashion our academic writing into a lively text. Many readers of *Thinking*

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\* There have also been three Nobel Prizes awarded for work in mechanism design and information economics, both of which are closely related to game theory: in 1996, to William Vickrey and James Mirrlees; in 2001, to George Akerlof, Michael Spence, and Joseph Stiglitz; and in 2007, to Leonid Hurwicz, Eric Maskin, and Roger Myerson.

*Strategically* gave us encouragement, advice, and criticism, all of which have helped us when writing *The Art of Strategy*. At the grave risk of omitting some, we must mention ones to whom we owe special thanks. Our coauthors on other related and unrelated book projects, Ian Ayres, Adam Brandenburger, Robert Pindyck, David Reiley, and Susan Skeath, generously gave us much useful input. Others whose influence continues in this new book include David Austen-Smith, Alan Blinder, Peter Grant, Seth Masters, Benjamin Polak, Carl Shapiro, Terry Vaughn, and Robert Willig. Jack Repcheck at W. W. Norton has been a constantly supportive, understanding, and perceptive editor for *The Art of Strategy*. Our manuscript editors, Janet Byrne and Catherine Pichotta, were generous to our faults. Every time you don't find a mistake, you should thank them.

We owe special thanks to Andrew St. George, book critic for the *Financial Times*. In choosing *Thinking Strategically* as a book he enjoyed reading most in the year 1991, he said: "it is a trip to the gym for the reasoning facilities" (*FT Weekend*, December 7/8, 1991). This gave us the idea of labeling some intriguing challenges we pose to the readers in this edition "Trips to the Gym." Finally, John Morgan, of the University of California, Berkeley, gave us a powerful incentive with the threat, "If you don't write a revision, I will write a competing book." And after we saved him the trouble, he helped us out with many ideas and suggestions.

Avinash Dixit  
Barry Nalebuff  
October 2007

## **Chapter 1: Ten Tales of Strategy**

We begin with ten tales of strategy from different aspects of life and offer preliminary thoughts on how best to play. Many of you will have faced similar problems in everyday life and will have reached the correct solution after some thought or trial and error. For others, some of the answers may be surprising, but surprise is not the primary purpose of the examples. Our aim is to show that such situations are pervasive, that they amount to a coherent set of questions, and that methodical thinking about them is likely to be fruitful.

In later chapters, we develop these systems of thought into prescriptions for effective strategy. Think of these tales as a taste of dessert before the main course. They are designed to whet your appetite, not fill you up.

### **#1. Pick a Number**

Believe it or not, we are going to ask you to play a game against us. We've picked a number between 1 and 100, and your goal is to guess the number. If you guess correctly on the first try, we'll pay you \$100.

Actually, we aren't really going to pay you \$100. It would be rather costly for us, especially since we want to give you some help along the way. But, as you play the game, we'd like you to imagine that we really are going to give you money, and we'll play the same way.

The chance of getting the number right on the first shot is quite low, only one in a hundred. So to improve your chances, we'll give you five guesses, and after each wrong guess, we'll also tell you if you are too high or too low. Of course, there's a bigger reward for getting the right answer quickly. If you guess correctly on the second try, you'll get \$80. On the third try, the payment is down to \$60, then \$40 for the fourth guess, and \$20 if you get the number on the fifth try. If it takes more than five guesses, the game is over and you get nothing.

Are you ready to play? We are, too. If you are wondering how to play a game with a book, it is a bit of a challenge, but not impossible. You can go to the [artofstrategy.info](http://artofstrategy.info) web site and play the game interactively. Or, we can anticipate how you might be playing the game and respond accordingly.

Is your first guess 50? That's the most common first guess and, alas for you, it's too high.

Might your second guess be 25? Following 50, that is what most folks do. Sorry, that's too low. The next step for many is 37. We're afraid that 37 is too low. What about 42? Too low, again.

Let's pause, take a step back, and analyze the situation. This is your fifth guess coming up and your last chance to take our money. You know the number is above 42 and less than 50. There are seven options: 43, 44, 45, 46, 47, 48, and 49. Which of those seven do you think it will be?

So far, you have been guessing in a way that divides the interval into two equal parts and picking the midpoint. This is the ideal strategy in a game where the number has been chosen at random.\* You are getting the most information possible from each guess and therefore will converge to the number as quickly as possible. Indeed, Microsoft CEO Steven Ballmer is said to have used this game as a job interview

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\* The technical term for this search strategy is minimizing the entropy.

question. For Ballmer the correct answer was 50, 25, 37, 42, . . . He was interested in seeing if the candidate approached the search problem in the most logical and efficient manner.

Our answer is a bit different. In Ballmer's problem, the number was picked at random, and so the engineer's strategy of "divide the set in two and conquer" was just right. Getting the most information from each guess minimizes the expected number of guesses and therefore leads you to get the most money. In our case, however, the number was *not* picked at random. Remember that we said that we were playing this game as if we actually had to pay you the money. Well, no one is reimbursing us for money that, hypothetically, we would have to pay you. And as much as we like you for having bought our book, we like ourselves even more. We'd rather keep the money than give it to you. So we deliberately picked a number that would be hard for you to find. Think about it for a moment—would it have made any sense for us to have picked 50 as the number? That would have cost us a fortune.

The key lesson of game theory is to put yourself in the other player's shoes. We put ourselves in your shoes and anticipated that you would guess 50, then 25, then 37, then 42. Understanding how you would play the game allowed us to greatly decrease the chance that you would guess our number and thus reduce how much we'd have to pay out.

In explaining all of this to you before the game is over, we've given you a big hint. So now that you understand the real game you are playing, you've got one last guess, for \$20. What number do you pick?

49?

Congratulations. To us, not you. You've fallen right into our trap again. The number we picked was forty-eight. Indeed, the whole speech about picking a number that was hard to find according to the split-the-interval rule was further designed to mislead you. We wanted you to pick 49 so that our choice of forty-eight would remain safe. Remember our objective is not to give you money.

To beat us at that game, you had to be one step ahead of us. You would have had to think, "They want us to pick 49, so I'm going to pick 48." Of course, if we had thought you would have been so clever, then we would have picked 47 or even 49.

The larger point of this game is not that we are selfish professors or cunning tricksters. Rather, the point is to illustrate as cleanly as possible what makes something a game: you have to take into account the objectives and strategies of the other players. When guessing a number picked at random, the number isn't trying to hide. You can take the engineer's mindset and divide the interval in two and do the best possible. But if you are playing a game, then you have to consider how the other player will be acting and how those decisions will influence your strategy.

## **#2. Winning by Losing**

We admit it: we watched *Survivor*. We would never have made it on the island. If we hadn't starved first, the others would surely have voted us off for being eggheads. For us, the challenge was trying to predict how the game would play out. We weren't surprised when the pudgy nudist Richard Hatch outwitted, outplayed, and outlasted his rivals to become the first champion of the CBS series and earn the million-dollar prize. He was gifted in his ability to act strategically without appearing to be strategic.

Hatch's most cunning ploy was in the last episode. The game was down to three players. Richard's two remaining rivals were 72-year-old retired Navy SEAL Rudy Boesch and 23-year-old river guide Kelly Wiglesworth. For their final challenge, the three

of them had to stand on a pole with one hand on the immunity idol. The last one standing would go into the finals. And just as important, the winner would get to choose his or her opponent in the finals.

Your first impression might be that this was just a physical endurance contest. Think again. All three players understood that Rudy was the most popular of the three. If Rudy made it to the finals, Rudy would likely win. Richard's best hope was to go against Kelly in the finals.

There were two ways that could happen. One is that Kelly would win the pole-standing competition and pick Richard. The other is that Richard would win and pick Kelly. Richard could count on Kelly picking him. She was also aware of Rudy's popularity. Her best hope of winning was to get to the finals against Richard.

It would seem that if either Richard or Kelly won the final challenge, each would pick the other as his or her opponent. Hence Richard should try to stay in the game, at least until Rudy had fallen off. The only problem is that Richard and Rudy had a long-standing alliance. If Richard won the challenge and didn't pick Rudy, that would have turned Rudy (and all Rudy's friends) against Richard, and this could have cost him the victory. One of the great twists of *Survivor* is that the ousted contestants vote to determine the final winner. One has to be very careful how one disposes of rivals.

From Richard's perspective, the final challenge could go one of three ways:

- (i) Rudy wins. Rudy then picks Richard, but Rudy would be the likely victor.
- (ii) Kelly wins. Kelly would be smart enough to know her best hope was to eliminate Rudy and go against Richard.
- (iii) Richard wins. If he picks Rudy to go on, Rudy beats him in the finals. If he picks Kelly to go on, Kelly might beat him because Richard would lose the support of Rudy and his many friends.

Comparing these options, Richard does best by losing. He wants Rudy eliminated, but it is better if Kelly does the dirty work for him. The smart money was on Kelly winning the challenge. She had won three of the previous four and as an outdoors guide was in the best shape of the three.

As a bonus, throwing the game saved Richard the trouble of standing on a pole under a hot sun. Early in the competition, host Jeff Probst offered a slice of orange to anyone willing to call it quits. Richard stepped off the pole and took the orange.

After 4 hours and 11 minutes, Rudy fumbled when shifting his stance, let go of the immunity idol, and lost. Kelly picked Richard to go on to the finals. Rudy cast the swing vote in his favor, and Richard Hatch became the first *Survivor* champion.

With the benefit of hindsight it may all

Throughout the book, you'll find these asides, which contain what we call a "Trip to the Gym." These trips take a look at more advanced elements of the game that we glossed over. For example, Richard could have tried to wait and see who dropped out first. If Kelly fell early, Richard might have preferred to beat Rudy and choose Kelly than to let Rudy win and have to go against Rudy in the finals. He might also have been concerned that Kelly would be savvy enough to do the same calculation and drop out early. The next chapters will show you how to use a more systematic approach to solve a game. The end goal is to help change the way you approach strategic situations, recognizing that you won't always have time to analyze every possible option.

seem easy. What makes Richard's play so impressive is that he was able to anticipate all the different moves before they happened.\* In chapter 2, we'll provide some tools to help you anticipate the way a game will play out and even give you a chance to have a go at another *Survivor* game.

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\* Richard would have done well to anticipate the consequences of not paying taxes on his \$1 million winnings. On May 16, 2006, he was sentenced to 51 months in prison for tax evasion.

## #6. Thinning Strategically

Cindy Nacson-Schechter wanted to lose weight. She knew just what to do: eat less and exercise more. She knew all about the food pyramid and the hidden calories in soft drinks. Still, nothing had worked. She had gained forty pounds with the birth of her second child and it just wasn't coming off.

That's why she accepted ABC's offer to help her lose weight. On December 9, 2005, she came into a photographer's studio on Manhattan's West Side, where she found herself changing into a bikini. She hadn't worn a bikini since she was nine, and this wasn't the time to start again.

The setup felt like backstage at the *Sports Illustrated* swimsuit issue shoot. There were lights and cameras everywhere, and all she had on was a tiny lime-green bikini. The producers had thoughtfully placed a hidden space heater to keep her warm. Snap. Smile. Snap. Smile. What in the world was she thinking? Snap.

If things worked out as she hoped, no one would ever see these pictures. The deal she made with ABC *Primetime* was that they would destroy the pictures if she lost 15 pounds over the next two months. They wouldn't help her in any way. No coach, no trainer, no special diets. She already knew what she had to do. All she needed was some extra motivation and a reason to start today rather than tomorrow.

Now she had that extra motivation. If she didn't lose the promised weight, ABC would show the photos and the videos on primetime television. She had already signed the release giving them permission.

Fifteen pounds in two months was a safe amount to lose, but it wouldn't be a cakewalk. There was a series of holiday parties and Christmas dinners. She couldn't risk waiting until the New Year. She had to start now.

Cindy knew all about the dangers of being overweight—the increased risk of diabetes, heart attack, and death. And yet that wasn't enough to scare her into action. What she feared more than anything was the possibility that her ex-boyfriend would see her hanging out of a bikini on national TV. And there was little doubt that he would watch the show. Her best friend was going to tell him if she failed.

Laurie Edwards didn't like the way she looked or how she felt. It didn't help that she worked part-time tending bar, surrounded by hot twenty-somethings. She had tried Weight Watchers, South Beach, Slim-Fast, you name it. She was headed in the wrong direction and needed something to help her change course. When she told her girlfriends about the show, they thought it was the stupidest thing she'd ever done. The cameras captured that "what am I doing?" look on her face and a lot more.

Ray needed to lose weight, too. He was a newlywed in his twenties but looked closer to forty. As he walked the red carpet in his racing swimsuit, it wasn't a pretty picture. Click. Smile. Click.

He wasn't taking any chances. His wife wanted him to lose weight and was willing to help. She offered to diet with him. Then she took the plunge. She changed into a bikini, too. She wasn't as overweight as Ray, but she wasn't bikini-ready, either.

Her deal was different from Cindy's. She didn't have to weigh in. She didn't even have to lose weight. The pictures of her in a bikini would only be shown if Ray didn't lose the weight.

For Ray, the stakes had been raised even higher. He was either going to lose the weight or his wife.

All together, four women and one couple bared their soles and much more in front of the cameras. What were they doing? They weren't exhibitionists. The ABC producers had carefully screened them out. None of the five wanted to see these photos appear on TV, and none of them expected they ever would.

They were playing a game against their future selves. Today's self wants the future self to diet and exercise. The future self wants the ice cream and the television. Most of the time, the future self wins because it gets to move last. The trick is to change the incentives for the future self so as to change its behavior.

In Greek mythology, Odysseus wanted to hear the Sirens' songs. He knew that if he allowed his future self to listen to their song, that future self would sail his ship into the rocks. So he tied his hands—literally. He had his crew bind his hands to the mast (while plugging their own ears). In dieting, this is known as the empty-fridge strategy.

Cindy, Laurie, and Ray went one step further. They put themselves in a bind that only dieting would get them out of. You might think that having more options is always a good thing. But thinking strategically, you can often do better by cutting off options. Thomas Schelling describes how the Athenian General Xenophon fought with his back against an impassable ravine. He purposefully set himself up so that his soldiers had no option of retreat.<sup>1</sup> Backs stiffened, they won.

Similarly, Cortés scuttled his ships upon arrival in Mexico. This decision was made with the support of his troops. Vastly outnumbered, his six hundred soldiers decided that they would either defeat the Aztecs or perish trying. The Aztecs could retreat inland, but for Cortés's soldiers there was no possibility of desertion or retreat. By making defeat worse, Cortés increased his chance of victory and indeed conquered.\*

What worked for Cortés and Xenophon worked for Cindy, Laurie, and Ray. Two months later, just in time for Valentine's Day, Cindy had lost 17 pounds. Ray was down 22 pounds and two belt loops. While the threat was the motivator to get them started, once they got going, they were doing it for themselves. Laurie lost the required 15 pounds in the first month. She kept on going and lost another 13 in month two. Laurie's 28 pounds translated into two dress sizes and over 14 percent of her body weight. Her friends no longer think the ABC show was a stupid idea.

At this point, you shouldn't be surprised to know that one of us was behind the show's design.<sup>2</sup> Perhaps we should have called this book *Thinning Strategically* and sold many more copies. Alas, not, and we return to study these types of strategic moves in chapter 6.

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\* Cortés was also helped by the Aztecs' misconception that he was Quetzalcoatl, a fair-skinned god.

## #8. Mix Your Plays

Apparently Takashi Hashiyama has trouble making decisions. Both Sotheby's and Christie's had made attractive offers to be the auction house for the sale of his company's \$18 million art collection. Rather than choose one over the other, he suggested the two of them play a game of Rock Paper Scissors to determine the winner. Yes, Rock Paper Scissors. Rock breaks scissors, scissors cuts paper, and paper covers rock.

Christie's chose scissors and Sotheby's chose paper. Scissors cut paper and so Christie's won the assignment and a nearly \$3 million commission. With the stakes so high, could game theory have helped?

The obvious point is that in this type of game, one can't be predictable. If Sotheby's had known that Christie's would be playing scissors, then they would have chosen rock. No matter what you choose, there is something else that beats it. Hence it is important that the other side can't predict your play.

As part of their preparation, Christie's turned to local experts, namely the kids of their employees who play the game regularly. According to eleven-year-old Alice, "Everybody knows you always start with scissors." Alice's twin sister, Flora, added her perspective: "Rock is way too obvious, and scissors beats paper. Since they were beginners, scissors was definitely the safest."<sup>3</sup>

Sotheby's took a different tack. They thought this was simply a game of chance and hence there was no room for strategy. Paper was as good as anything else.

What is interesting here is that both sides were half right. If Sotheby's picked its strategy at random—with an equal chance of rock, scissors, or paper—then whatever Christie's did would have been equally good. Each option has a one-third chance of winning, a one-third chance of losing, and a one-third chance of a tie.

But Christie's didn't pick at random. Thus Sotheby's would have done better to think about the advice Christie's would likely get and then play to beat it. If it's true that everyone knows you start with scissors, Sotheby's should have started with Bart Simpson's favorite, good old rock.

In that sense, both players got it half wrong. Given Sotheby's lack of strategy, there was no point in Christie's efforts. But given Christie's efforts, there would have been a point to Sotheby's thinking strategically.

In a single play of a game, it isn't hard to choose randomly. But when games get repeated, the approach is trickier. Mixing your plays does not mean rotating your strategies in a predictable manner. Your opponent can observe and exploit any systematic pattern almost as easily as he can exploit an unchanging repetition of a single strategy. It is *unpredictability* that is important when mixing.

It turns out most people fall into predictable patterns. You can test this yourself online where computer programs are able to find the pattern and beat you.<sup>4</sup> In an effort to mix things up, players often rotate their strategies too much. This leads to the surprise success of the "avalanche" strategy: rock, rock, rock.

People are also too influenced by what the other side did last time. If both Sotheby's and Christie's had opened with scissors, there would have been a tie and a rematch. According to Flora, Sotheby's would expect Christie's to play rock (to beat their scissors). That should lead Sotheby's to play paper and so Christie's should stick with scissors. Of course, that formulaic approach can't be right, either. If it were, Sotheby's could then play rock and win.

Imagine what would happen if there were some known formula that determined who would be audited by the IRS. Before you submitted a tax return, you could apply the formula to see if you would be audited. If an audit was predicted, but you could see a way to “amend” your return until the formula no longer predicted an audit, you probably would do so. If an audit was unavoidable, you would choose to tell the truth. The result of the IRS being completely predictable is that it would audit exactly the wrong people. All those audited would have anticipated their fate and chosen to act honestly, while those spared an audit would have only their consciences to watch over them. When the IRS audit formula is somewhat fuzzy, everyone stands some risk of an audit; this gives an added incentive for honesty.

The importance of randomized strategies was one of the early insights of game theory. The idea is simple and intuitive but needs refinement to be useful in practice. It is not enough for a tennis player to know that he should mix his shots between the opponent’s forehand and backhand. He needs some idea of whether he should go to the forehand 30 percent or 64 percent of the time and how the answer depends on the relative strengths of the two sides. In chapter 5 we develop methods to answer such questions.

We’d like to leave you with one last commentary. The biggest loser in the Rock Paper Scissors game wasn’t Sotheby’s; it was Mr. Hashiyama. His decision to deploy Rock Paper Scissors gave each of the two auction houses a 50 percent chance of winning the commission. Instead of letting the two contestants effectively agree to split the commission, he could have run his own auction. Both firms were willing, even eager, to lead the sale with a 12 percent commission.\* The winning house would be the one willing to take the lowest fee. Do I hear 11 percent? Going once, going twice, . . .

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\* The standard commission is 20 percent on the first \$800,000 and 12 percent thereafter. Mr. Hashiyama’s four paintings sold for a combined \$17.8 million, suggesting a total commission of \$2.84 million.

## Case Study: Multiple Choice

We think almost everything in life is a game, even things that might not seem that way at first. Consider the following question from the GMAT (the test given to MBA applicants).

Unfortunately, issues of copyright clearance have prevented us from reproducing the question, but that shouldn't stop us. Which of the following is the correct answer?

- a)  $4\pi$  sq. inches
- b)  $8\pi$  sq. inches
- c) 16 sq. inches
- d)  $16\pi$  sq. inches
- e)  $32\pi$  sq. inches

Okay, we recognize that you're at a bit of a disadvantage not having the question. Still, we think that by putting on your game-theory hat you can still figure it out.

## Case Discussion

The odd answer in the series is c. Since it is so different from the other answers, it is probably not right. The fact that the units are in square inches suggests an answer that has a perfect square in it, such as  $4\pi$  or  $16\pi$ .

This is a fine start and demonstrates good test-taking skills, but we haven't really started to use game theory. Think of the game being played by the person writing the question. What is that person's objective?

He or she wants people who understand the problem to get the answer right and those who don't to get it wrong. Thus wrong answers have to be chosen carefully so as to be appealing to folks who don't quite know the answer. For example, in response to the question: How many feet are in a mile, an answer of "Giraffe," or even  $16\pi$ , is unlikely to attract any takers.

Turning this around, imagine that 16 square inches really is the right answer. What kind of question might have 16 square inches as the answer but would lead someone to think  $32\pi$  is right? Not many. People don't often go around adding  $\pi$  to answers for the fun of it. "Did you see my new car—it gets  $10\pi$  miles to the gallon." We think not. Hence we can truly rule out 16 as being the correct solution.

Let's now turn to the two perfect squares,  $4\pi$  and  $16\pi$ . Assume for a moment that  $16\pi$  square inches is the correct solution. The problem might have been what is the area of a circle with a radius of 4? The correct formula for the area of a circle is  $\pi r^2$ . However, the person who didn't quite remember the formula might have mixed it up with the formula for the circumference of a circle,  $2\pi r$ . (Yes, we know that the circumference is in inches, not square inches, but the person making this mistake would be unlikely to recognize this issue.)

Note that if  $r = 4$ , then  $2\pi r$  is  $8\pi$ , and that would lead the person to the wrong answer of b. The person could also mix and match and use the formula  $2\pi r^2$  and hence believe that  $32\pi$  or e was the right answer. The person could leave off the  $\pi$  and come up with 16 or c, or the person could forget to square the radius and simply use  $\pi r$  as the area, leading to  $4\pi$  or a. In summary, if  $16\pi$  is the correct answer, then we can tell a

plausible story about how each of the other answers might be chosen. They are all good wrong answers for the test maker.

What if  $4\pi$  is the correct solution (so that  $r = 2$ )? Think now about the most common mistake, mixing up circumference with area. If the student used the wrong formula,  $2\pi r$ , he or she would still get  $4\pi$ , albeit with incorrect units. There is nothing worse, from a test maker's perspective, than allowing the person to get the right answer for the wrong reason. Hence  $4\pi$  would be a terrible right answer, as it would allow too many people who didn't know what they were doing to get full credit.

At this point, we are done. We are confident that the right answer is  $16\pi$ . And we are right. By thinking about the objective of the person writing the test, we can suss out the right answer, often without even seeing the question.

Now, we don't recommend that you go about taking the GMAT and other tests without bothering to even look at the questions. We appreciate that if you are smart enough to go through this logic, you most likely know the formula for the area of a circle. But you never know. There will be cases where you don't know the meaning of one of the answers or the material for the question wasn't covered in your course. In those cases, thinking about the testing game may lead you to the right answer.

## Notes

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<sup>1</sup> See Thomas Schelling, *Arms and Influence* (New Haven, CT: Yale University Press, 1966), 45; and Xenophon, *The Persian Expedition* (London: Penguin, 1949), 136–37, 236.

<sup>2</sup> The show, *Life: The Game*, aired on March 16, 2006. A DVD is available for purchase at [www.abcnewsstore.com](http://www.abcnewsstore.com) as “PRIMETIME: Game Theory: 3/16/06.” A sequel, where this threat was contrasted with positive reinforcement, aired on December 20, 2006 and is available as “PRIMETIME: Basic Instincts – Part 3 – Game Theory: 12/20/06.”

<sup>3</sup> Our quotes are from the *New York Times* coverage of the story, May 29, 2005.

<sup>4</sup> One online option is Perry Friedman’s AI algorithm at <http://chappie.stanford.edu/cgi-bin/roshambot>. It placed sixteenth in the second international RoShamBo programming competition; [www.cs.ualberta.ca/~darse/rsbpc.html](http://www.cs.ualberta.ca/~darse/rsbpc.html). For readers looking to brush up their skills, we recommend Douglas Walker and Graham Walker’s *The Official Rock Paper Scissors Strategy Guide* (New York: Simon & Schuster, 2004) and a visit to [www.worldrps.com](http://www.worldrps.com).